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An analytical study of the effect of convection heat transfer on the sublimation of a frozen semi-infinite porous medium

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Abstract—Analytical solutions are developed for the effect of convection heat transfer on the temperature, moisture concentration, pressure and sublimation front location in a sublimating frozen semi-infinite moist porous medium. The mass transfer in the dried region is modeled by the application of Darcy's and Fick's laws coupled with an additional model for the pressure in this region. Results are presented which show the influence of convective heat transfer on the sublimation front position to be significant only for those combinations of boundary conditions, initial conditions and thermal–physical properties that yield high rates of sublimation for the no convection heat transfer case.

INTRODUCTION

THE SUBLIMATION process for freeze-drying various products has been widely used for the past decades in the food, medical and chemical industries. The main advantages of the sublimation dehydration are that the shape and quality of the heat-sensitive materials can be maintained. Owing to the simultaneous and non-linear complexities of the phase change problems of the porous media associated with the heat and mass transfer, only a few solutions are obtained under the various simplified assumptions [1–5].

The effect of convection heat transfer during sublimation dehydration and drying of moist porous media has received little attention in the literature. Generally investigators assume that the heat transfer through a porous medium is due to conduction heat transfer only. Since there is a vapor flow through the dried region which moves from the sublimating/vaporizing material toward the outer boundary, energy is convected from the sublimation or drying interface. Therefore, the energy available for the phase change at the interface is reduced to some extent. Recently, Haber *et al.* [6] obtained an exact solution for the effect of convection heat transfer on the drying rate of a moist porous half-space by using a model based upon Darcy's law for describing the vapor transfer in the dried region.

This paper proposes a model which describes the vapor flow through the dried region of a sublimating substance by including the effect of convection heat transfer and by the application of both Darcy's law and Fick's law for the moisture transfer analysis. In addition, a model to describe the pressure distribution within the dried region is included based upon the work of refs. [7, 8]. The present formulation leads to an exact analytical solution for the transient model of the sublimation process. Since it is of more interest to know the effect of heat convection on the sublimation rate, this work presents figures to show the variation of the interface position with various significant parameters and discusses the physical trends involved in the sublimation process.

STATEMENT OF THE PROBLEM

A semi-infinite frozen porous medium is exposed to an environment where the pressure is maintained below the triple point of the bounded substance, the temperature is higher than the initial temperature of the frozen porous medium, and the vapor concentration is maintained at a sufficiently low value. Figure 1 shows a semi-infinite frozen porous medium where the temperature and mass content are initially constant throughout the medium. At time greater than zero the temperature, vapor concentration and pressure of the surface at $x = 0$ are maintained at constant values and the sublimation process begins. Since the vapor transfers outwardly, two regions (i.e. dried region and frozen region) are formed and are separated by a distinct moving interface located by $x = s(t)$. The temperature, vapor concentration and pressure at the interface are unknown but are assumed to have constant values which will be determined later. The moisture content is kept at its initial value and no moisture movement occurs in the frozen region; however, temperature gradients exist in this region which will affect the heat flux at the sublimation interface and thus the sublimation rate. We therefore consider only the heat conduction equation in this region. In the dried region vapor concentration flows occur as a result of the interactions of temperature, vapor concentration and pressure gradients. For

NOMENCLATURE

b	heat convection parameter
C	molar concentration of moisture
C_0	molar concentration of frozen bounded substance
c_p	specific heat
c_{pw}	specific heat of vapor
$\bar{C}$	non-dimensional molar concentration in dried region, $(C - C_s)/(C_v - C_s)$
$\bar{C}_0$	non-dimensional initial content of frozen bounded substance, $\omega C_0/C_3$
$\bar{C}_s$	non-dimensional surface molar concentration of vapor, C_s/C_3
$\bar{C}_v$	non-dimensional interface molar concentration of vapor, C_v/C_3
K	effective thermal conductivity
K_{21}	thermal conductivity ratio, K_2/K_1
l	latent heat of sublimation
L	non-dimensional latent heat of sublimation, $l/(R_0 T_3)$
Lu	Luikov moisture diffusivity, α_m/α_2
Lu_p	Luikov filtration diffusivity, α_p/α_2
M_m	molecular weight of bounded substance
P	pressure
P_s	total pressure at surface
$\bar{P}$	non-dimensional pressure in dried region, $(P - P_s)/(P_v - P_s)$
$\bar{P}_s$	non-dimensional surface pressure, P_s/P_3
$\bar{P}_v$	non-dimensional interface pressure, P_v/P_3
Q	non-dimensional parameter, $C_3 M_m \alpha_2 l/(T_3 K_1)$
R	non-dimensional gas constant, $C_3 R_0 T_3/P_3$
R_0	universal gas constant
$s(t)$	position of sublimation interface
t	time
T	temperature
T_0	initial temperature
$\dot{W}$	mass flow rate of vapor
x	space coordinate
$Z(\eta)$	transformation variable, $P(\eta) + \beta C(\eta)$.

Greek symbols

α	effective thermal diffusivity
α_m	effective moisture diffusivity
α_p	filtration motion diffusion coefficient of vapor
α_{21}	thermal diffusivity ratio, α_2/α_1
β	$(\alpha_m - \varepsilon\alpha_p)/\kappa$
Δ	non-dimensional permeability, $\varepsilon\kappa P_3/[(\varepsilon\alpha_p - \alpha_m)C_3]$ or $-\varepsilon P_3/(\beta C_3)$
ε	porosity
η	similarity variable, $x/(2\sqrt{(\alpha_2 t)})$
θ_1	non-dimensional temperature in frozen region, $(T_1 - T_0)/(T_s - T_0)$
θ_2	non-dimensional temperature in dried region, $(T_2 - T_0)/(T_s - T_0)$
θ_0	non-dimensional initial temperature, T_0/T_3
θ_s	non-dimensional surface temperature, T_s/T_3
θ_v	non-dimensional interface temperature, T_v/T_3
κ	permeability
λ	non-dimensional position of sublimation interface, $s(t)/(2\sqrt{(\alpha_2 t)})$
ρ	density
ω	volume fraction of frozen bounded substance.

Subscripts

1	frozen region, $s(t) < x < \infty$
2	dried region, $0 < x < s(t)$
3	at triple point of bounded substance
s	at surface, $x = 0$
v	at interface of sublimation, $x = s(t)$.

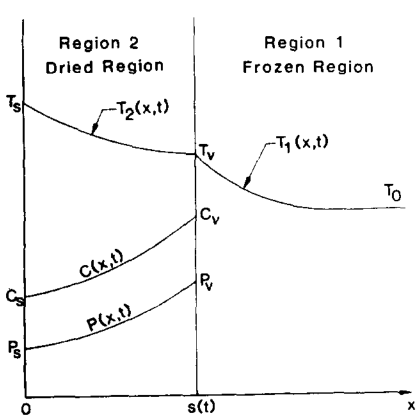

FIG. 1. Schematic description for the sublimation problem.

completeness, we include the heat convection term due to this vapor flow in the heat equation.

To formulate the theoretical model of the sublimation problem, the following assumptions are made.

(1) The one-dimensional transfers of heat and mass are considered. The radiative heat transfer within the medium is assumed negligible.

(2) The porous medium is assumed to be composed of very small solid particles of the same size. Also, the medium is isotropic, homogeneous and rigid. Thermal expansion of the porous medium can be neglected.

(3) The interface of sublimation is sharply thin and separates the porous medium into two regions. The frozen region maintains its initial uniform content of the bounded substance while the dried region is filled with vapor only [9].

(4) The velocities of vapor motion are large enough as compared to the sublimation interface velocities

that the vapor flow rate throughout the dried region is assumed to be approximately equal to the vapor flow rate rate at the sublimation interface at any given time.

(5) Both Darcy's and Fick's laws for mass transfer are valid through the dried region. The Soret and Dufour effects are assumed negligible [10, 11].

(6) The thermophysical properties remain constant but may be different for different regions.

(7) The non-condensable gas in the dried region is exhausted by the vapor flow and only the vapor exists in this region [9].

(8) The latent heat of sublimation is constant and the Clapeyron equation provides a relation among the interface temperature, pressure and the latent heat of sublimation. Also, the ideal gas law is assumed to be valid at the interface.

(9) The temperature, pressure and concentration at the interface are in equilibrium and constant but depend upon the initial and boundary conditions, Clapeyron equation [4] and equation of state.

(10) The temperature of the dried porous body is equal to that of the vapor flowing through it. Accordingly the vapor is considered to be superheated and recondensation may be neglected.

From the above assumptions the sublimation process can be formulated by the following equations:

$$(\rho c_p)_1 \frac{\partial T_1(x,t)}{\partial t} = K_1 \frac{\partial^2 T_1(x,t)}{\partial x^2}, \; s(t) < x < \infty \quad (1)$$

$$(\rho c_p)_2 \frac{\partial T_2(x,t)}{\partial t} = K_2 \frac{\partial^2 T_2(x,t)}{\partial x^2} + c_{pw} \dot{W} \frac{\partial T_2(x,t)}{\partial x},$$

$$0 < x < s(t) \quad (2)$$

$$\varepsilon \frac{\partial C(x,t)}{\partial t} = \alpha_m \frac{\partial^2 C(x,t)}{\partial x^2} + \kappa \frac{\partial^2 P(x,t)}{\partial x^2}, \; 0 < x < s(t) \quad (3)$$

$$\frac{\partial P(x,t)}{\partial t} = \alpha_p \frac{\partial^2 P(x,t)}{\partial x^2}, \; 0 < x < s(t) \quad (4)$$

where equations (1) and (2) describe the temperature distribution in the frozen and dried regions, respectively. Equations (3) and (4), based on assumption (5) and the Luikov system [7, 8], describe the concentration and pressure fields in the dried region, respectively. It is noted that the second term on the right-hand side of equation (2) represents the heat convection due to vapor flow. According to assumption (4), the mass flow rate $\dot{W}$, which has the same value throughout the dried region at any given time, can be expressed as

$$\dot{W} = \omega M_m C_0 \frac{ds(t)}{dt}. \quad (5)$$

We also note that the first and second terms on the right-hand side of equation (3) represent Fick's and Darcy's laws, respectively. The boundary and initial conditions are

$$T_1(x,0) = T_1(\infty,t) = T_0 \quad (6)$$

$$T_2(0,t) = T_s \quad (7)$$

$$C(0,t) = C_s. \quad (8)$$

$$P(0,t) = P_s. \quad (9)$$

At the sublimation interface the conditions are

$$T_1(s,t) = T_2(s,t) = T_v \quad (10)$$

$$C(s,t) = C_v \quad (11)$$

$$P(s,t) = P_v \quad (12)$$

where T_v, C_v and P_v are the interface temperature, molar concentration and pressure, respectively, and are assumed to be constants but unknowns.

The Clapeyron equation which relates the latent heat of sublimation to the interface conditions, from equation (8) in ref. [4], is

$$\frac{C_v T_v}{C_3 T_3} = \exp\left[\frac{l}{R_0}\left(\frac{1}{T_3} - \frac{1}{T_v}\right)\right]. \quad (13)$$

By applying the ideal gas law at the sublimation interface, we have

$$P_v = C_v R_0 T_v. \quad (14)$$

The energy and moisture balances at the interface yield

$$-K_2 \frac{\partial T_2(s,t)}{\partial x} + K_1 \frac{\partial T_1(s,t)}{\partial x} = \omega C_0 M_m l \frac{ds(t)}{dt} \quad (15)$$

$$\alpha_m \frac{\partial C(s,t)}{\partial x} + \kappa \frac{\partial P(s,t)}{\partial x} = \left[\omega C_0 - \varepsilon C(s,t)\right] \frac{ds(t)}{dt}. \quad (16)$$

SOLUTION OF THE PROBLEM

We introduce the dimensionless similarity variable

$$\eta = \frac{x}{2\sqrt{(\alpha_2 t)}} \quad (17)$$

into equations (1)–(16) and define a new variable $Z(\eta)$ as given in refs. [12, 13] which decouples equations (3) and (4)

$$Z(\eta) = P(\eta) + \beta C(\eta) \quad (18)$$

where

$$\beta = \frac{\alpha_m - \varepsilon \alpha_p}{\kappa}. \quad (19)$$

The location of the interface is assumed to be given by

$$s(t) = 2\lambda\sqrt{(\alpha_2 t)} \quad (20)$$

where λ is an unknown constant to be determined during the solution.

With the introduction of the new variables η and λ, we note that the dried region corresponds to $0 < \eta < \lambda$, and the frozen region corresponds to $\lambda < \eta < \infty$. The problem is now transformed to the following set of ordinary differential equations with variable coefficients subject to the transformed boundary and interface conditions

$$\frac{d^2T_1(\eta)}{d\eta^2} + 2\frac{\alpha_2}{\alpha_1}\eta\frac{dT_1(\eta)}{d\eta} = 0,\ \lambda < \eta < \infty \quad (21)$$

$$\frac{d^2T_2(\eta)}{d\eta^2} + 2(b\lambda + \eta)\frac{dT_2(\eta)}{d\eta} = 0,\ 0 < \eta < \lambda \quad (22)$$

$$\frac{d^2P(\eta)}{d\eta^2} + 2\frac{\alpha_2}{\alpha_p}\eta\frac{dP(\eta)}{d\eta} = 0,\ 0 < \eta < \lambda \quad (23)$$

$$\frac{d^2Z(\eta)}{d\eta^2} + 2\varepsilon\frac{\alpha_2}{\alpha_m}\eta\frac{dZ(\eta)}{d\eta} = 0,\ 0 < \eta < \lambda \quad (24)$$

where the heat convection parameter b is

$$b = \frac{\omega M_m c_{pw} C_0 \alpha_2}{K_2} \quad (25)$$

which will indicate the existence and effect of the heat convection. When $b = 0$, equation (22) naturally reduces to the heat conduction equation only.

The boundary conditions are

$$T_1(\eta = \infty) = T_0 \quad (26)$$

$$T_2(\eta = 0) = T_s \quad (27)$$

$$T_1(\eta = \lambda) = T_2(\eta = \lambda) = T_v \quad (28)$$

$$C(\eta = 0) = C_s \quad (29)$$

$$C(\eta = \lambda) = C_v \quad (30)$$

$$P(\eta = 0) = P_s \quad (31)$$

$$P(\eta = \lambda) = P_v \quad (32)$$

$$Z(\eta = 0) = P_s + \beta C_s \quad (33)$$

$$Z(\eta = \lambda) = P_v + \beta C_v. \quad (34)$$

The energy and moisture balance equations are

$$-K_2\frac{dT_2(\eta = \lambda)}{d\eta} + K_1\frac{dT_1(\eta = \lambda)}{d\eta} = 2\omega C_0 M_m \alpha_2 l\lambda \quad (35)$$

$$\alpha_m\frac{dC(\eta = \lambda)}{d\eta} + \kappa\frac{dP(\eta = \lambda)}{d\eta} = 2\alpha_2\lambda(\omega C_0 - \varepsilon C_v). \quad (36)$$

The above equations (21)–(24) with boundary conditions (26)–(34) can be solved exactly. Once obtaining the solutions for $P(\eta)$ and $Z(\eta)$, the solution for $C(\eta)$ is yielded by equation (18). Here we present the solutions as follows:

$$T_1(\eta) = T_0 + (T_v - T_0)\frac{\mathrm{erfc}(\sqrt{(\alpha_2/\alpha_1)}\eta)}{\mathrm{erfc}(\sqrt{(\alpha_2/\alpha_1)}\lambda)} \quad (37)$$

$$T_2(\eta) = T_s + (T_v - T_s)\frac{\mathrm{erfc}(b\lambda) - \mathrm{erfc}(\eta + b\lambda)}{\mathrm{erfc}(b\lambda) - \mathrm{erfc}(\lambda + b\lambda)} \quad (38)$$

$$P(\eta) = P_s + (P_v - P_s)\frac{\mathrm{erf}(\sqrt{(\alpha_2/\alpha_p)}\eta)}{\mathrm{erf}(\sqrt{(\alpha_2/\alpha_p)}\lambda)} \quad (39)$$

$$C(\eta) = \left[\frac{P_v - P_s}{\beta} + (C_v - C_s)\right]\frac{\mathrm{erf}(\sqrt{(\varepsilon\alpha_2/\alpha_m)}\eta)}{\mathrm{erf}(\sqrt{(\varepsilon\alpha_2/\alpha_m)}\lambda)} + C_s + \frac{(P_s - P_v)}{\beta}\frac{\mathrm{erf}(\sqrt{(\alpha_2/\alpha_p)}\eta)}{\mathrm{erf}(\sqrt{(\alpha_2/\alpha_p)}\lambda)}. \quad (40)$$

Upon substituting equations (37)–(40) into the interface equations (35) and (36), we obtain two transcendental equations. These equations along with equations (13) and (14) give four relations for the unknown interface variables T_v, C_v, P_v and λ.

These equations are

$$\frac{K_2(T_s - T_v)}{\mathrm{erfc}(b\lambda) - \mathrm{erfc}(\lambda + b\lambda)}e^{-(\lambda + b\lambda)^2} - \frac{\sqrt{(\alpha_2/\alpha_1)}K_1(T_v - T_0)}{\mathrm{erfc}(\sqrt{(\alpha_2/\alpha_1)}\lambda)}e^{-(\alpha_2/\alpha_1)\lambda^2} = \sqrt{\pi}\,\omega C_0 M_m \alpha_2 l\lambda \quad (41)$$

and

$$\frac{\sqrt{(\varepsilon\alpha_2/\alpha_m)}}{\mathrm{erf}(\sqrt{(\varepsilon\alpha_2/\alpha_m)}\lambda)}\left[\frac{P_v - P_s}{\beta} + (C_v - C_s)\right]e^{-(\varepsilon\alpha_2/\alpha_m)\lambda^2} + \frac{\sqrt{(\alpha_2\alpha_p)}}{\mathrm{erf}(\sqrt{(\alpha_2/\alpha_p)}\lambda)}\frac{(P_s - P_v)}{\beta}e^{-(\alpha_2/\alpha_p)\lambda^2} + \frac{(\kappa/\alpha_m)\sqrt{(\alpha_2/\alpha_p)}(P_v - P_s)}{\mathrm{erf}(\sqrt{(\alpha_2/\alpha_p)}\lambda)}e^{-(\alpha_2/\alpha_p)\lambda^2} = \sqrt{\pi}\frac{\alpha_2}{\alpha_m}\lambda(\omega C_0 - \varepsilon C_v). \quad (42)$$

Using the non-dimensional parameters defined in the nomenclature, the solution is now transformed to the following non-dimensional form:

$$\theta_1 = \frac{\theta_v - \theta_0}{\theta_s - \theta_0}\frac{\mathrm{erfc}(\sqrt{\alpha_{21}}\eta)}{\mathrm{erfc}(\sqrt{\alpha_{21}}\lambda)},\ \lambda < \eta < \infty \quad (43)$$

$$\theta_2 = 1 + \frac{\theta_v - \theta_s}{\theta_s - \theta_0}\frac{\mathrm{erfc}(b\lambda) - \mathrm{erfc}(\eta + b\lambda)}{\mathrm{erfc}(b\lambda) - \mathrm{erfc}(\lambda + b\lambda)},\ 0 < \eta < \lambda \quad (44)$$

$$\bar{P} = \frac{\mathrm{erf}(\eta/\sqrt{Lu_p})}{\mathrm{erf}(\lambda/\sqrt{Lu_p})},\ 0 < \eta < \lambda \quad (45)$$

$$\bar{C} = \left(\frac{\Delta(\bar{P}_s - \bar{P}_v)}{\varepsilon(\bar{C}_v - \bar{C}_s)} + 1\right)\frac{\mathrm{erf}(\sqrt{(\varepsilon/Lu)}\eta)}{\mathrm{erf}(\sqrt{(\varepsilon/Lu)}\lambda)} + \frac{\Delta}{\varepsilon}\left(\frac{\bar{P}_v - \bar{P}_s}{\bar{C}_v - \bar{C}_s}\right)\frac{\mathrm{erf}(\eta/\sqrt{Lu_p})}{\mathrm{erf}(\lambda/\sqrt{Lu_p})},\ 0 < \eta < \lambda. \quad (46)$$

The interface conditions become

$$K_{21}\left(\frac{\theta_s-\theta_v}{\theta_v-\theta_0}\right)\frac{e^{-(\lambda+b\lambda)^2}}{\mathrm{erfc}(b\lambda)-\mathrm{erfc}(\lambda+b\lambda)} - \sqrt{\alpha_{21}}\frac{e^{-\alpha_{21}\lambda^2}}{\mathrm{erfc}(\sqrt{\alpha_{21}}\lambda)} = \frac{\sqrt{\pi}\bar{C}_0 Q\lambda}{\theta_v-\theta_0} \qquad (47)$$

and

$$\frac{\sqrt{(Lu/\varepsilon)}[\Delta(\bar{P}_s-\bar{P}_v)+\varepsilon(\bar{C}_v-\bar{C}_s)]e^{-\varepsilon\lambda^2/Lu}}{\mathrm{erf}(\lambda\sqrt{(Lu/\varepsilon)})} + \frac{\Delta(\bar{P}_v-\bar{P}_s)\sqrt{Lu_p}\,e^{-\lambda^2/Lu_p}}{\mathrm{erf}(\lambda/\sqrt{Lu_p})} = \sqrt{\pi}\,\lambda(\bar{C}_0-\varepsilon\bar{C}_v). \qquad (48)$$

The Clapeyron equation becomes

$$\bar{C}_v\theta_v = \exp[L(1-1/\theta_v)] \qquad (49)$$

and the ideal gas law becomes

$$\bar{P}_v = \bar{C}_v R\theta_v. \qquad (50)$$

The non-dimensional constant λ and non-dimensional temperature θ_v, pressure $\bar{P}_v$, molar concentration $\bar{C}_v$ at the interface are then obtained by numerically solving the simultaneous equations (47)–(50). Once the interface constants are known, equations (43)–(46) readily yield the exact solution to the sublimation problem.

RESULTS AND DISCUSSION

To better understand the effects of convective heat transfer on the sublimation process, a porous medium and a sublimating substance of a sand bed and ice, respectively, are assumed for illustration. Since the sublimation rate is of prime interest during the sublimation dehydration process, results are now presented which illustrate the effect of selected parameters on the non-dimensional interface position λ. The speed of the sublimation interface is proportional to the parameter λ; thus, larger values of λ result in higher speeds of sublimation. Based upon refs. [7, 9 14, 15], the sublimation interface parameter, λ, is plotted for a reference set of data as a function of the convection parameter, b. On the figures presented in this study, only the parameters having values different from the reference values are indicated. The selected reference values are: $K_{21} = 1.0$, $\alpha_{21} = 1.0$, $\Delta = 0.1$, $Lu_p = 300$, $\bar{C}_0 = 20$, $Lu = 0.1$, $\bar{C}_s = 0.2$, $\theta_0 = 0.9$, $\theta_s = 1.0$, $\bar{P}_s = 0.2$, $\varepsilon = 0.38$, $L = 22.5$, $R = 0.987$, $Q = 3.6 \times 10^{-6}$. It is noted from ref. [7] that the values of Lu_p range from 100 to 1000; a value of $Lu_p = 300$ is used here for illustration only.

The results of this study show that the sublimation interface parameter, λ, decreases with increasing convection parameter, b. This is due to the fact that during the sublimation process, the sublimated mass carries energy from the sublimation interface as it flows through the dried region to the outer boundary. In the following figures the convection parameter, b, varies from a zero value which corresponds to the case of no convection heat transfer to a value of 3 which corresponds to strong convective effects.

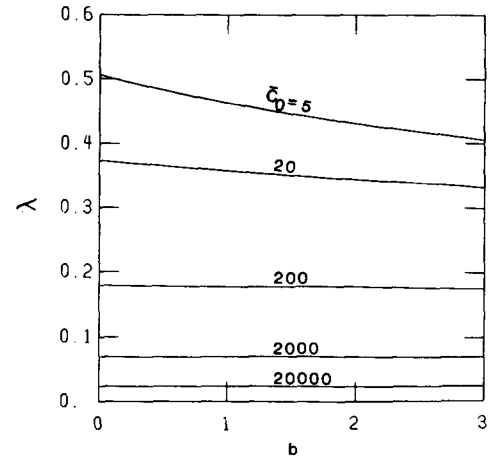

FIG. 2. Effect of convection parameter on dimensionless interface position for various values of initial content of frozen bounded substance.

Figure 2 shows the variation of the dimensionless interface position, λ, with the convective parameter, b, for several values of the non-dimensional initial content of the frozen bounded substance, $\bar{C}_0$. A comparison between cases $b = 3$ and 0 indicates a reduction of 20% in λ for $\bar{C}_0 = 5$ and only 0.006% for $\bar{C}_0 = 2 \times 10^4$. A low value of $\bar{C}_0$ means that the amount of sublimating material at the interface is low which may be a result of a low value of the volume fraction of the frozen bounded substance or a low value of the C_0/C_3 ratio. As $\bar{C}_0$ increases more energy must be absorbed at the sublimation interface which results in a lower sublimation speed. As a result, the convection term in the dried region energy balance would have a small value regardless of the value of b. Thus, the effect of convection heat transfer in the dried region is significant only for low values of the non-dimensional initial content.

Figure 3 illustrates the effect of the convection parameter, b, on the dimensionless interface position, λ, for various values of the dimensionless permeability, Δ. Comparing the results for $b = 3$ and 0 shows a 11% reduction in λ for Δ of 0.1 and a 27% reduction in λ for Δ of 2. As the permeability increases, the resistance to flow through the dried porous region decreases. Thus the convective mass flow through the dried layer increases and carries larger amounts of convective energy from the sublimation interface. This results in significant reductions in the interface position. Also, the rate of increase of the non-dimensional sublimation front with the permeability is reduced as the permeability increases.

Figure 4 shows the variation of the dimensionless interface position, λ, with the convective parameter,

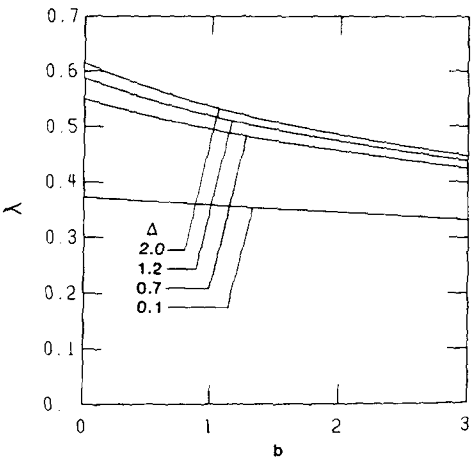

FIG. 3. Effect of convection parameter on dimensionless interface position for various values of dimensionless permeability.

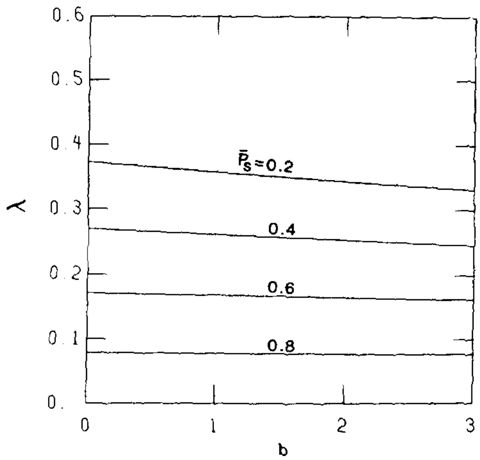

FIG. 5. Effect of convection parameter on dimensionless interface position for various values of dimensionless surface pressure.

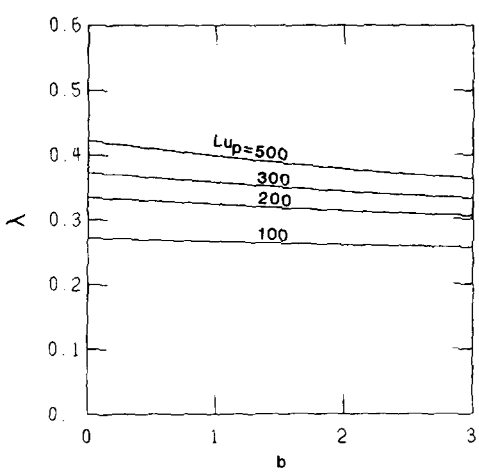

FIG. 4. Effect of convection parameter on dimensionless interface position for various values of the Luikov filtration diffusivity.

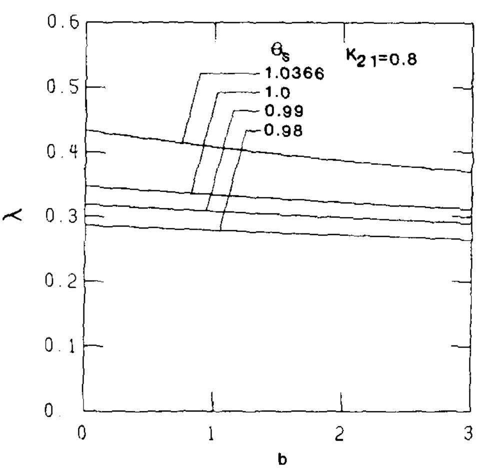

FIG. 6. Effect of convection parameter on dimensionless interface position for various values of dimensionless surface temperature.

b, for various values of the Luikov filtration diffusivity, Lu_p. The comparison of the results for $b = 3$ and 0 indicates a reduction of 6% in λ for Lu_p of 100 and 14% for Lu_p of 500. Since Lu_p is a measure of the speed of propagation of pressure waves through the material, one would expect an increased sublimation rate, more material sublimated, and an increased effect of the convection heat transfer as Lu_p increases.

Figure 5 illustrates the variation of the dimensionless interface position, λ, with the convective parameter, b, for various dimensionless surface pressures, $\bar{P}_s$. In general, as the surface pressure is decreased, the sublimation process speeds up. However, this figure shows that convection plays a more important role in the sublimation process for lower values of the surface pressure. For instance, the dimensionless interface position is reduced by 11% as b increases from 0 to 3 for $\bar{P}_s$ of 0.2 but is reduced by only 1.5% for the same range of b for $\bar{P}_s$ of 0.8.

The effect of the dimensionless convective parameter, b, on the dimensionless interface position, λ, for various values of dimensionless surface temperature, θ_s, for a thermal conductivity ratio, K_{21}, of 0.8 is shown in Fig. 6. For a surface temperature θ_s of 1.0366 (i.e. 10°C above the triple point temperature of water), the interface position, λ, decreases by 15% as the convective parameter, b, increases from 0 to 3. For θ_s of 0.98, λ decreases by 8% over the same range of b. Since the sublimation interface position is sensitive to the surface temperature, inclusion of the convective heat transfer term results in significant effects on λ.

Figure 7 shows the effect of the dimensionless convection parameter, b, on the dimensionless interface position, λ, for several values of non-dimensional initial temperature, θ_0. These results indicate that increasing the non-dimensional initial temperature results in a bounded substance that is easier to

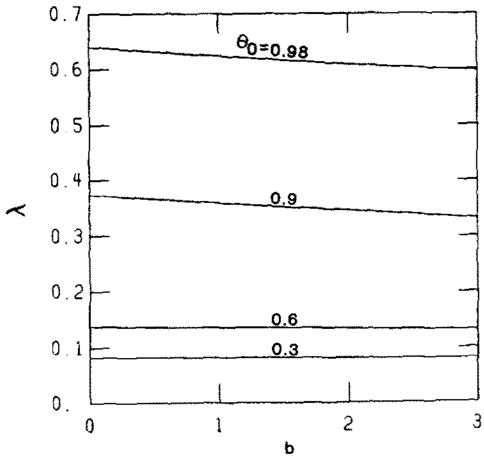

FIG. 7. Effect of convection parameter on dimensionless interface position for various values of dimensionless initial temperature.

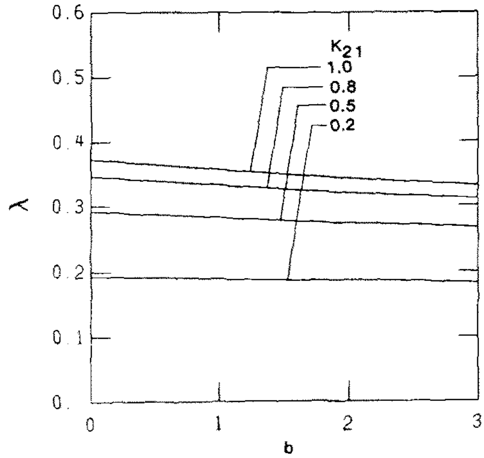

FIG. 9. Effect of convection parameter on dimensionless interface position for various values of thermal conductivity ratio.

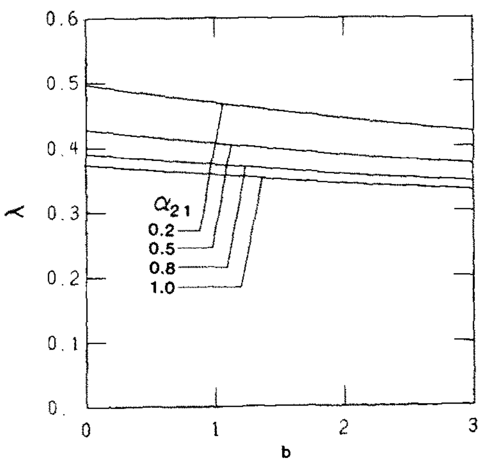

FIG. 8. Effect of convection parameter on dimensionless interface position for various values of thermal diffusivity ratio.

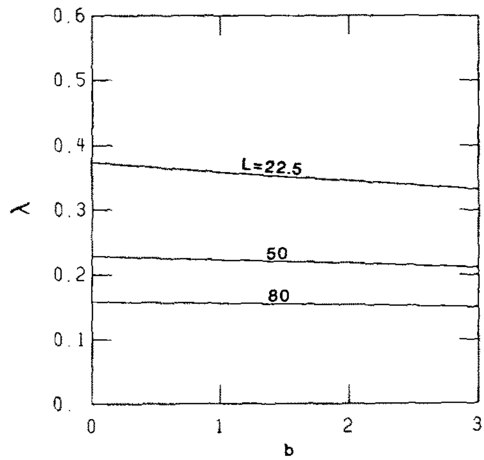

FIG. 10. Effect of convection parameter on dimensionless interface position for various values of dimensionless latent heat.

sublimate; therefore, a faster speed of sublimation occurs for larger values of θ_0. For the small values of the initial temperature the sublimation speed is low and the effect of the convection parameter, b, is small. The effect of the convection parameter is more significant for the higher initial temperature cases.

Figure 8 shows the effect of the dimensionless convective parameter, b, on the dimensionless interface position, λ, for various values of the thermal diffusivity ratio, α_{21}. Over the range of the convective parameters studied, the influence of the thermal diffusivity ratio on the interface position is small. As b varied from 0 to 3 the difference in percent reduction of λ as α_{21} was varied from 0.2 to 1.0 was only 4%.

Figure 9 illustrates the effect of the dimensionless convective parameter, b, on the dimensionless interface position, λ, for several values of the thermal conductivity ratio, K_{21}. As the convective parameter, b, is increased from 0 to 3 the dimensionless interface position decreases by 5% for K_{21} of 0.2 and by 11% for K_{21} of 1.0. In general as K_{21} increases, the applied heat flux at the sublimation interface increases. This results in increased rates of sublimation and larger amounts of convective energy carried from the sublimation interface which causes the decreased sublimation rate as K_{21} increases.

Figure 10 shows the effect of the dimensionless convective parameter, b, on the dimensionless sublimation interface position, λ, for several values of the dimensionless latent heat, L. A comparison of the results indicates that a reduction of 11% in λ occurs for L of 22.5 and 5% for L of 80. This trend is explained by the fact that materials with lower sublimation latent heats sublimate faster and have larger mass flow rates for convective heat transfer from the sublimation interface.

Results for other parameters such as Lu and $\bar{C}_s$ have also been obtained. Variations in these parameters have negligible effects on the non-dimensional interface position, λ, for all values of the convection heat transfer parameter, b.

CONCLUSION

According to the above discussion, the following conclusions may be drawn.

(1) Convection heat transfer is significant for sublimation parameters which result in large sublimation speeds. To judge whether to include the heat convection term, one needs only to consider those cases having large sublimation speeds for the no heat convection case, $b = 0$.

(2) Variations in the physical parameters ($\bar{C}_0$, Δ), the boundary conditions ($\bar{P}_s, \theta_s$), and the initial temperature parameter, θ_0, influenced the dimensionless interface position, λ, more than did the thermal-moisture parameters K_{21}, α_{21}, Lu_p and L.

(3) The effect of convection heat transfer during the sublimation process is pronounced only for low values of the non-dimensional initial content of frozen bounded substance, $\bar{C}_0$ (i.e. a small value of the volume fraction of the frozen bounded substance, ω). In most applications of the sublimation dehydration process, ω is not small and the effect of heat convection may be neglected.

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ETUDE ANALYTIQUE DE L'EFFET DE CONVECTION THERMIQUE SUR LA SUBLIMATION D'UN MILIEU SEMI-INFINI POREUX GELE

Résumé—On développe des solutions analytiques de l'effet de convection thermique sur la température, la concentration d'humidité, la pression et la position du front de sublimation dans un milieu poreux gelé et semi-infini. Le transfert massique dans la région asséchée est modélisé par l'application des lois de Darcy et de Fick couplées avec un modèle supplémentaire pour la pression dans cette région. Des résultats sont présentés qui montrent que l'influence du transfert de chaleur convectif sur la position du front de sublimation est sensible seulement pour la combinaison des conditions limites, conditions initiales et propriétés thermo-physiques qui conduisent à des vitesses élevées de sublimation pour le cas sans convection thermique.

EINE ANALYTISCHE UNTERSUCHUNG DES EINFLUSSES VON KONVEKTIVEM WÄRMEÜBERGANG AUF DIE SUBLIMATION EINES GEFRORENEN HALBUNENDLICHEN PORÖSEN MEDIUMS

Zusammenfassung—Analytische Lösungen werden für den Einfluß des konvektiven Wärmeübergangs auf die Temperatur, die Feuchtekonzentration, den Druck und den Ort der Sublimationsfront in einem sublimierenden, gefrorenen halbunendlichen feucht-porösen Medium entwickelt. Der Stofftransport in der getrockneten Region wird durch die Anwendung der Gesetze von Darcy und Fick modelliert, gekoppelt mit einem zusätzlichen Modell für den Druck in dieser Region. Es werden Ergebnisse präsentiert, die zeigen, daß der Einfluß des konvektiven Wärmeübergangs auf den Ort der Sublimationsfront nur für jene Kombinationen von Rand- und Anfangsbedingungen bedeutend ist, die eine hohe Sublimationsgeschwindigkeit für den Fall des nicht-konvektiven Wärmeübergangs ergeben.

АНАЛИТИЧЕСКОЕ ИССЛЕДОВАНИЕ ВЛИЯНИЯ КОНВЕКТИВНОГО ТЕПЛОПЕРЕНОСА НА СУБЛИМАСИЮ ЗАМОРОЖЕННОЙ ПОЛУБЕСКОНЕЧНОЙ ПОРИСТОЙ СРЕДЫ

Аннотация—Получены аналитические решения с учетом влияния конвективного теплопереноса на температуру, концентрацию влаги, давление и положение фронта сублимации в сублимирующей замороженной полубесконечной влажной пористой среде. Массоперенос в осушенной области моделируется законами Дарси и Фика с привлечением дополнительной модели для давлений в этой области. Представленные результаты показывают, что влияние конвективного теплопереноса на положение фронта сублимации существенно только для таких комбинаций граничных, начальных условий и теплофизических свойств, для которых скорость сублимации достаточно велика и в случае отсутствия конвективного теплопереноса.